Elementary derivation of Black Hole area-entropy relation

Consider a black hole with radius R and mass M.
Entropy can be correlated with information so we will consider the case when one bit of information falls into the black hole.

A photon of wavelength R can be said to have one bit of information.i.e either it exists or not. A photon of larger wavelength has more information regarding the possibility it will move around the black hole and not fall into it, while a photon with smaller wavelength has additional information about the angle at which it enters the black hole, etc.

The energy of the photon is given as

$$E = \frac{hc}{\lambda} = \frac{hc}{R}$$

where h is Planck's constant and c is the speed of light, and corresponding mass,

$$m = \frac{E}{c^2} = \frac{h}{Rc}$$

When this photon falls into the black hole, its mass increases by $\delta M = m$.
From the Schwarzschild radius expression,

$$R = \frac{2GM}{c^2},$$

where Rs is the radius of event horizon, for a change in mass by $\delta M = m$, the radius changes by

$$\delta R = \frac{2G\delta M}{c^2} = \frac{2Gh}{Rc^3}$$

Rearranging,

$$R\delta R = \frac{2Gh}{c^3}$$

we know that area $A \propto R^2$ which means $\delta A \propto R\delta R$.
From the above two equations,

$$\delta A = \frac{2Gh}{c^3}$$

i.e, the change in area of event horizon when one bit of information falls into the black hole is a constant independent of the radius or mass of the black hole. Therefore, the entropy of black hole is directly proportional to the area of its event horizon.

The actual entropy-area relation is the famous Bekenstein-Hawking entropy formula,

$$S = \frac{\pi A c^3}{2hG}$$